

## APPENDIX 5

### The case of the $N = 4n + 1$ order

**Preliminary observation** - To preserve a lattice with  $n + 1$  cells with coefficients  $a_i$ , with  $0 \leq i \leq n$ , as defined for the  $4n + 3$  order, we present the case of the  $4n + 5$  order.

Let  $L_i$ ,  $i = 0, \dots, 3$  be the polyphase components of a null ISI  $4n + 3$  order  $L(z)$  filter. It is possible to associate, with this filter, a lattice built as in Figure 8a of the patent with a set of coefficients  $\alpha_i$ ,  $i = 0, \dots, n$ . We have  $L_2(z) = \hat{L}_1(z)$  and  $L_3(z) = \hat{L}_0(z)$ .

The filter  $F(z)$  whose components  $F_i$ ,  $i=0, \dots, 3$  are defined by

$$(56) \quad F_0(z) = L_0(z) ,$$

$$(57) \quad F_1(z) = z^{-1} \hat{L}_0(z) ,$$

$$(58) \quad F_2(z) = L_1(z) ,$$

$$(59) \quad F_3(z) = \hat{L}_1(z) ,$$

is a null ISI,  $4n + 5$  order linear phase filter. Furthermore, the Theorem 2 in Appendix 2 of the patent establishes the fact that such filters are necessarily obtained in this way. Of course, the synthesis of such a filter is done so as to optimize the coefficients  $\alpha_i$ ,  $i = 0, \dots, n$  to obtain the right frequency specifications for the filter  $F(z)$ , not for the filter  $L(z)$  ! This therefore leads to the obtaining of diagrams for sending and reception associated with  $F(z)$  from the lattice built with the coefficients  $\alpha_i$  and the associated inverted lattice.

The type I polyphase breakdown applied to this filter is expressed by the following expression:

$$F(z) = L_0(z^4) + z^{-1}[z^{-4}\hat{L}_0(z^4)] + z^{-2}L_1(z^4) + z^{-3}\hat{L}_1(z^4) \quad (60)$$

It can be ascertained that, for the sending part, this equation may be expressed by the lattice diagram of Figure 20 wherein a delay is applied. This delay is given by  $z^{-3}$  at output of  $F(z)$ .

For the reception part as in the case of the  $4n + 3$  order, it is possible to make a 4-lattice system that ensures the totality of the properties sought.

Similarly, simplifications appear if we consider the system known as the back-to-back system. The input signal of the reception filter can then be written as follows:

$$Y(z) = [z^{-3}L_0(z^4) + z^{-8}[z^{-4}\hat{L}_0(z^4)] + z^{-5}L_1(z^4) + z^{-6}\hat{L}_1(z^4)]X(z^4) \quad (61)$$

To recover the input signal, apart from a delay, we then implement the device of Figure 21.

Referring to Figure 21, the signals after this decimation are, from top to bottom, given by:

$$\begin{aligned} Y(z) \downarrow_4 &= z^{-2}\hat{L}_0(z)X(z), & z^{-1}Y(z) \downarrow_4 &= z^{-1}L_0(z)X(z), \\ z^{-2}Y(z) \downarrow_4 &= z^{-2}\hat{L}_1(z)X(z), & z^{-3}Y(z) \downarrow_4 &= z^{-2}L_1(z)X(z) \end{aligned} \quad (62)$$

From top to bottom, the signals at input of the lattice are respectively given by:

$$z^{-2}[\hat{L}_0(z) + \hat{L}_1(z)]X(z) \text{ et } z^{-2}[L_0(z) - L_1(z)]X(z).$$

Thus, as in the case of the  $4n + 3$  order but this time with a processing delay of  $n + 2$  samples, we have an output signal which, for  $g = 1/(2\gamma)$  is identical to that of the input.